Journal of Civil Engineering and Environmental Technology
p-ISSN: 2349-8404; e-ISSN: 2349-879X; Volume 10, Issue 1; January-March, 2023, pp. 22-27

http://www.krishisanskriti.org/Publication.html

# Active Control of a Cantilever Beam under Static Loading using Piezoelectric Patches

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**Abstract—***Structures that are subjected to static and dynamic loadings they respond and it is very necessary to accurately sense and control their static and dynamic behavior to avoid failure. Active vibration control (AVC) is a method to attenuate the vibrations of the structures by providing an equal amount of vibrations in the opposite direction. In this paper, the simulated study is conducted to accurately sense and control the static deflection of the cantilever beam by using piezoelectric patches. The accurate prediction of the static deflection of the cantilever beam and its control through piezoelectric patches is the novelty of the present work. Mostly, the finite element (FE) model of the beam is developed to predict its dynamic behavior. In this regard, static deflection of the cantilever beam is predicted by FE approach along with state space method and a direct output feedback control algorithm is developed to attenuate the static deflection of the cantilever beam. The static deflection of the beam in an open loop and closed loop is predicted and is compared to find the reduction in the static deflection. It is found that the direct output feedback controller attenuates the static deflection of the cantilever beam by 45.06 %.*

**Keywords:** *Piezoelectric transducers, static loading, Finite element model*

## INTRODUCTION

Piezoelectric transducers are widely used to sense and control the response of the structures when subjected under static and dynamic loadings.

Active control of a composite beam under dynamic loading was carried out using piezoelectric patches by Huang et al. A linear quadratic regulator (LQR) controller was implemented to attenuate the dynamic response of the beam [1]. The efficiency of an LQR controller was examined by Aktas et al to attenuate the vibration of cantilever beam. They allocated piezoelectric sensors and actuators consecutively at the fixed end, the middle part, and the free end to optimally locate and control the dynamic response of the beam [2]. An accelerometer is used to simulate the dynamic response of a cantilever beam and AVC was carried out using LQR controller by Mirafzal et al. to determine the optimal time delay feedback [3]. A proportional integral derivative (PID) controller was used to attenuate the dynamic behaviour of a cantilever beam by Kumar et al. using piezoelectric patches [4]. Active vibration control of cantilever beam was carried out by Fei. They bonded PZT patches near the fixed end and controlled dynamic response using PID compensator, strain rate feedback by implementing using xPC Target real-time system[5]. AVC using a LQR controller on a cantilever beam using piezoelectric transducers was carried out by Zhang et al.[6]. A PID controller was designed by Khot et al. for AVC of a cantilever beam using piezoelectric patches[7]. The optimal location of piezoelectric patches on a cantilever plate was determined by Qiu et al. for AVC of dynamic response using a positive position feedback and proportional-derivative (PD) control[8]. Piezoelectric patches were used by Wu et al. to improve the damping capability of a cantilever beam by using a mode control method [9]. SISO system for a cantilever was modeled by Chhabra et al. to predict the dynamic response and AVC was implemented by using PID controller, proportional controller and pole placement technique [10]. A strain gauge sensor was used to measure the dynamic response of the composite cantilever beam. The response was controlled by using a piezoelectric patch as an actuator and by implementing a PID controller [11]. $H_\infty$ and μ-synthesis controllers are used by Sahin et al. to suppress the dynamic response of a flat aluminium-cantilever fin using piezoelectric patches [12]. A simulation study was performed on a flexible cantilever plate by Qiu et al. and also described the optimal location of piezoelectric patches by considering the performance of LQR controller [13].

It can be concluded from the above literature that dynamic response of the structure was sensed and controlled by using piezoelectric patches. However, prediction of the behaviour of structures and control of the response, under static loading is missing in the literature. In this regard, in this paper, an algorithm is developed to predict the response of a cantilever beam under static loading. However, a direct output feedback control algorithm is also developed to attenuate the static deflection of a cantilever beam.

## NUMERICAL STUDY

In this section, the finite element modeling of the smart structure is carried out. An algorithm is also developed with piezoelectric transducers to sense and actively control the static deflection of the beam.

### Finite element model of the beam embedded with piezoelectric transducers

Verma et al. developed the finite element model of cantilever beam embedded with piezoelectric transducers by using a Euler Bernoulli's beam theory[14]. The beam element has same transverse deflection throughout the cross-section [15]. Figure 1 shows the beam affixed with piezoelectric patches on the fixed end.

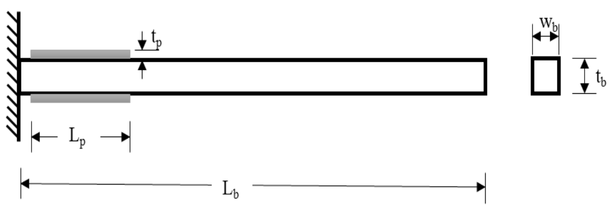


**Figure 1: Shows cantilever beam embedded with piezoelectric patches.**

The stiffness matrix for the beam element is given by [15],

$$\left[K_b\right] = \int_0^L \left[B\right]^T \left[B\right] AEdx \tag{1}$$

$$[K_b] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{2}$$

Where, L is the length of the beam element, E is the elastic modulus of the beam element and I is the moment of inertia of the beam element.

The mass matrix of the beam element is given by [15],

$$\left[M_b\right] = \int_0^L \rho\left(\left[N_i\right]^T\left[N_i\right]\right) Adx \tag{3}$$

$$\left[M_b\right] = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{4}$$

Where, ρ is the density of the beam element, A is the area of the cross-section of the beam.

When piezoelectric patches are coupled with the beam, it acts as an electro-elastic beam. Similarly, the expressions for piezoelectric patches can be derived[14].

The stiffness matrix of the piezoelectric transducer can be obtained as:

$$\left[K_p\right] = A_{uu}\int_0^L \left[B_u\right]\left[B_u\right]^T dx \tag{5}$$

$$A_{uu} = \frac{w_b C^E L_p}{3}\left(3t_b^2 t_p + 3t_b t_p^2 + t_p^3\right) \tag{6}$$

$$[K_p] = \frac{A_{uu}}{L_p^4}\begin{bmatrix} 12 & 6L_p & -12 & 6L_p \\ 6L & 4L_p^2 & -6L_p & 2L_p^2 \\ -12 & -6L_p & 12 & -6L_p \\ 6L_p & 2L_p^2 & -6L_p & 4L_p^2 \end{bmatrix} \tag{7}$$

The piezoelectric mass matrix can be obtained as:

$$\left[M_p\right] = \int_0^{L_p} \rho_p\left(\left[N_i\right]^T\left[N_i\right]\right) A_p dx \tag{8}$$

$$\left[M_p\right] = \frac{\rho_p A_p L_p}{420}\begin{bmatrix} 156 & 22L_p & 54 & -13L_p \\ 22L_p & 4L_p^2 & 13L_p & -3L_p^2 \\ 54 & 13L_p & 156 & -22L_p \\ -13L_p & -3L_p^2 & -22L_p & 4L_p^2 \end{bmatrix} \tag{9}$$

The one form of energy is converted into another form by using electro-mechanical coupling[16].

$$[K_{u\phi}] = \frac{A_{u\phi}}{L_p}\int_0^{L_p} [B_u]^T [B_\phi] dx \tag{10}$$

$$[K_{u\phi}] = \frac{A_{u\phi}}{L_p t_p}\begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 0 & 0 \\ -1 & 1 \end{bmatrix} \tag{11}$$

Where, $A_{u\phi} = -\frac{w_b e_{31} L_p}{2}\left(2t_b t_p + t_p^2\right)$

Now piezoelectric capacitance matrix can be defined as

$$[K_{\phi\phi}] = -A_p \epsilon_{33}^S L_p \int_0^{L_p} [B_\phi]^T [B_\phi] dx \tag{12}$$

$$[K_{\phi\phi}] = -\frac{A_p \epsilon_{33}^S L_p}{t_p}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{13}$$

Where $A_p$ is the area of piezoelectric patches, $\epsilon_{33}^S$ is dielectric permittivity at constant mechanical strain, and $e_{31}$ electric permittivity.

**State space model of the beam affixed with piezoelectric transducers**

The state space model of the beam is developed to predict the response of the beam under static loading. Taking structural physical displacement and structural physical velocity as state variables. Hence, state space equations can be written as:

$$\{\dot{u}_i\} = [A]\{u\} + [B]\{f\} \tag{14}$$

$$\{y\} = [C]\{u\} + [D]\{f\} \tag{15}$$

Where [A] is the system matrix, [B] is the input matrix, [C]is output matrix and, [D] is the direct transmission matrix, {u} are state variables, {f} is force vector and {y}is output vector.

$$\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_d \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ M^{-1} & -M^{-1}K_{u\phi}^{a} \end{bmatrix} \begin{Bmatrix} F^e \\ \phi_a \end{Bmatrix} \tag{16}$$

The output equation with displacement, velocity, acceleration, sensor output voltage and the charge accumulated on piezoelectric electrodes as outputs can be written as,

$$\begin{Bmatrix} u \\ \dot{u} \\ \ddot{u} \\ \phi_s \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -M^{-1}K & -M^{-1}C_d \\ S_{ca}K_{u\phi}^{s\,T} & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ M^{-1} & -M^{-1}K_{u\phi}^{a} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} F^e \\ \phi_a \end{Bmatrix} \tag{17}$$

Now comparing the equations (16) and (17) with (14) and (15), one can get

$$[A] = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_d \end{bmatrix} \& [B] = \begin{bmatrix} 0 & 0 \\ M^{-1} & -M^{-1}K_{u\phi}^{a} \end{bmatrix} \tag{18}$$

$$[C] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -M^{-1}K & -M^{-1}C_d \\ S_{ca}K_{u\phi}^{s\,T} & 0 \end{bmatrix} \& [D] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ M^{-1} & -M^{-1}K_{u\phi}^{a} \\ 0 & 0 \end{bmatrix} \tag{19}$$

**Develop a direct output feedback controller to attenuate the static deflection of the beam**

In this section, the active control of beam formulation under static loading is developed by using direct output feedback law. Suppression of vibrations of a cantilever beam was performed by Parameswaran et al. by using a DOFB controller[17]. The dynamic response of a cantilever beam embedded with piezoelectric patches was controlled by using DOFB [18].

The state space model of the beam-piezo system is,

$$\begin{Bmatrix} \dot{u}_1(t) \\ \dot{u}_2(t) \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_d \end{bmatrix} \begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ M^{-1} & -M^{-1}K_{u\phi}^{a} \end{bmatrix} \begin{Bmatrix} F^e \\ \phi_a \end{Bmatrix} \tag{20}$$

Direct output feedback law is given as[19],

$$\phi_a = -G_d\phi_s - G_v\dot{\phi}_s \tag{21}$$

Where $G_d$ and $G_v$ are the displacement and velocity gain respectively and $\phi_s$ is the sensor output voltage is given as,

$$\phi_s = S_{ca}K_{u\phi}^{s\,T}u \tag{22}$$

On expanding the equation.

$$\begin{Bmatrix} \dot{u}_1(t) \\ \dot{u}_2(t) \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_d \end{bmatrix} \begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} F^e + \begin{bmatrix} 0 \\ -M^{-1}K_{u\phi}^{a} \end{bmatrix} \phi_a \tag{23}$$

Substituting the value of $\phi_a$ from Equation (19) Into Equation (21), the following equation is obtained

On substituting the value of $\phi_s$ and $\dot{\phi}$s the following Equation can be obtained,

$$\begin{Bmatrix} \dot{u}_1(t) \\ \dot{u}_2(t) \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K_{new} & -M^{-1}C_{new} \end{bmatrix} \begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} F^e \tag{24}$$

Where,

$K_{new} = \left(K - K_{u\phi}^{a}G_dS_{ca}K_{u\phi}^{s\,T}\right)$ and $C_{new} = \left(C_d - K_{u\phi}^{a}G_vS_{ca}K_{u\phi}^{s\,T}\right)$ are the new stiffness and damping matrix with the control respectively.

Now substituting the new stiffness and damping matrices, one can get

$$\begin{Bmatrix} \phi_a \\ \ddot{u} \end{Bmatrix} = \begin{bmatrix} -G_dS_{ca}K_{u\phi}^{s\,T} & -G_vS_{ca}K_{u\phi}^{s\,T} \\ -M^{-1}K_{new} & -M^{-1}C_{new} \end{bmatrix} \begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} F^e \tag{25}$$

**METHODOLOGY**

In this section, sense and control the static deflection of cantilever beam using piezoelectric patches is discussed. A structure embedded with piezoelectric sensors and actuators is termed as *smart structure*[14]. Figure 2 shows the flow diagram to sense and attenuate the static deflection of the cantilever beam by using piezoelectric patches.

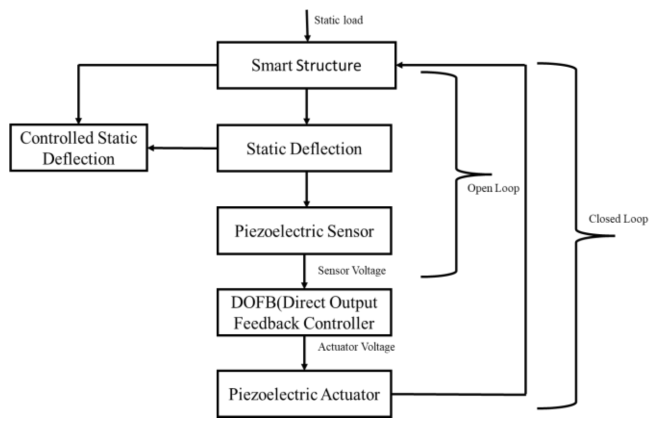


**Figure 2: Flowchart to sense and control the static deflection of beam using piezoelectric transducers.**

It can be seen from Figure 2, when the cantilever beam is subjected to static loading it undergoes static deflection. The deflection is predicted using a piezoelectric transducer and generates a voltage with reference to the static deflection of the beam. The beam embedded with piezoelectric sensor used to measure the response of the beam is termed as open loop model. The static response of the cantilever beam predicted by piezoelectric transducer is then fed to the direct output feedback (DOFB) controller. The controller manipulates the response and generates a voltage that is fed to the piezoelectric actuator and hence, forms a closed loop model.

## RESULT AND DISCUSSIONS

In this section, the simulation results of the sense and control the static deflection of a cantilever beam are discussed.

### Static deflection of the cantilever beam

The static deflection of the cantilever beam is predicted theoretically and numerically. The properties that are used in this simulation study is shown in Table 1.

**Table 1: Properties of cantilever beam and piezoelectric patch**

| Properties | Cantilever beam | Piezoelectric Patch |
|---|---|---|
| Length | 0.15 m | 0.061 m |
| Width | 0.045m | 0.035 m |
| Thickness | 0.0035 m | 0.0005 m |
| Elastic Modulus | 200 GPa | 23.3 GPa |
| Density | 8050 Kg/m$^3$ | 7800 Kg/m$^3$ |
| Dielectric Permittivity | - | -8.8678 (N-m/Vm$^2$) |
| Dielectric tensor at constant mechanical stress | - | 1.549×10$^{-8}$ F/m |

The deflection of cantilever beam is calculated by

$$\delta = \frac{PL^3}{3EI} \tag{26}$$

On the applied static load of 1N, static deflection of the cantilever beam calculated by equation () is 34.65×10$^{-06}$ m. The static deflection of the cantilever beam is also predicted using ABAQUS CAE as shown in Figure 3.

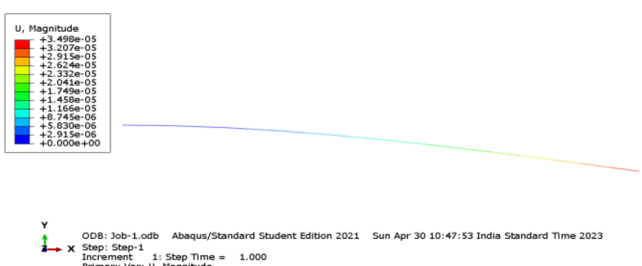


**Figure 3: Shows the static deflection of cantilever beam in ABAQUS CAE.**

It can be seen from Figure 3 the maximum static deflection is 34.98×10$^{-06}$ m and is on the free end of the cantilever beam.

### Sensing the static deflection of the cantilever beam using piezoelectric patch

The static deflection of the cantilever beam is predicted by using piezoelectric patch. The static response of the cantilever beam is in terms of voltage and is shown in Figure 4.

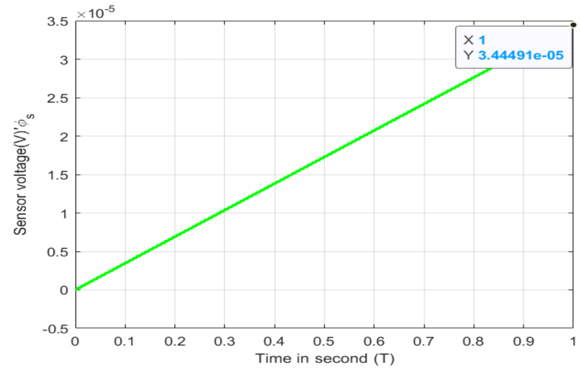


**Figure 4: Sensor output voltage in open loop model.**

It can be seen from Figure 4 the sensor output voltage produced on the application of static load is 34.449×10$^{-06}$ V. The deflection of the cantilever beam and piezoelectric sensor output voltage is shown in Table 2.

**Table 2: Shows comparison of static deflection and sensor voltage**

| S. No. | Applied Load (N) | Static deflection (μm) | Sensor Output Voltage (μV) |
|---|---|---|---|
| 1. | 1 | 34.98 | 34.449 |

It can be seen from Table 2 that the static deflection and sensor output voltage well corelates with each other.

### Control the static deflection of cantilever beam using piezoelectric patch

After sensing the static deflection of the cantilever beam, control the static deflection of cantilever beam using DOFB controller is discussed in this section. On implementing DOFB controller, at displacement feedback gain ($G_d$) and velocity feedback gain ($G_v$) 0 and 50 respectively, the sensor output voltage in open and closed loop is shown in Figure 5.

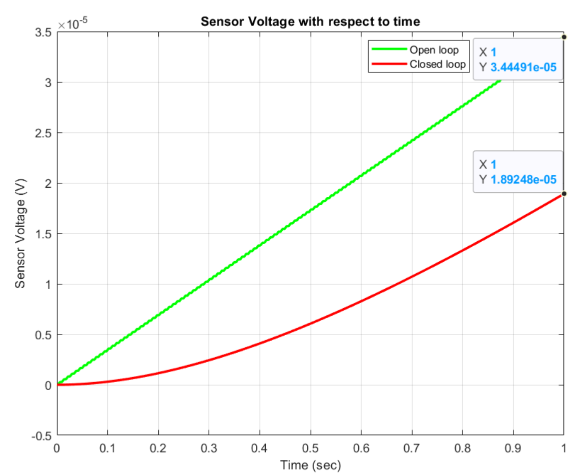


**Figure 5: shows sensor output voltage in open and closed loop**

It can be seen from Figure 5 that the static deflection of the cantilever beam is reduced from 34.449 µm to 18.924 µm. The actuator voltage required to control the static deflection of the cantilever beam is shown in Figure 6.

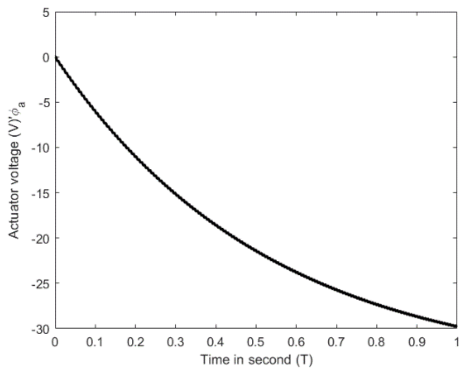


**Figure 6: Actuator voltage required for deflection control**

It can be seen from Figure 6 that the maximum actuator voltage required to control the static deflection of the cantilever beam is -29.84 V. At different values of velocity feedback gain the sensor output voltage in closed loop and actuator voltage is shown Table 3.

**Table 3: Shows actuator voltage at different velocity feedback gain.**

| S. No. | $G_d$ | $G_v$ | Sensor Voltage in closed loop (µV) | Actuator Voltage (V) |
|---|---|---|---|---|
| 1 | 0 | 0.1 | 34.46 | -0.12 |
| 2 | 0 | 1 | 34.13 | -0.74 |
| 3 | 0 | 10 | 30.82 | -7.01 |
| 4 | 0 | 20 | 27.21 | -13.97 |
| 5 | 0 | 25 | 25.52 | -17.21 |
| 6 | 0 | 30 | 23.95 | -20.23 |
| 7 | 0 | 50 | 18.92 | -29.84 |

It can be seen from Table 3 the sensor output voltage in closed loop is reducing with increase in the velocity feedback gain. Hence it can the deflection of the beam is reduced by 45.06 %.

## CONCLUSIONS

It is necessary to predict the dynamic parameters (natural frequencies and mode shapes) of the structures in control of the structures under dynamic loading. However, for the control under static loading it is necessary to predict the static deflection of the structure. In this work, the static deflection of the beam is predicted theoretically and numerically, it is found that the deflection well corelates with each other. The static deflection of the cantilever beam is predicted using PZT transducers. FE model of the beam embedded with PZT transducers is developed to sense and control the static deflection of cantilever beam. On sensing the static deflection using PZT transducers, it is found that the deflection and sensor output voltage well corelates with each. In this paper, a direct output feedback controller is developed to attenuate the static deflection of the cantilever beam. A simulated study is performed by giving different velocity feedback gains and sensor output voltage in closed loop is observed. From the results, it can be seen that at velocity feedback gain of 50 the static deflection of the cantilever beam is reduced by 46.05 %.

## ACKNOWLEDGEMENTS

This work was supported by a grant from the Indian Space Research Organization (ISRO).

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